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ABSTRACT

The basic scope of this study is to investigate the learning of mathematical concepts by primary grade students whose native language is other than English (Spanish). The efforts of the investigators in the study and the analysis of students' behaviors are to be concentrated in the areas of addition and subtraction operations, concept of number, concept of numeration, and the language involved in mathematical concepts. A summary of the tasks accomplished by the end of three months is presented along with a projection of the activities which are expected to be completed during the period of the time between the three- and six-month reports. A first-grade classroom from a predominantly Spanish-speaking elementary school in Dade County, Florida and one from a predominantly English-speaking elementary school were selected for purposes of the study. A summary description of the participating student sample at each of the schools is given along with a discussion of the initial testing procedures. A survey of the current Dade County Systems Mathematics curriculum was completed and a detailed list of objectives is presented. The pilot test form of a teacher survey of the perceived difficulty level of systems objectives is also included. (Author/SN)

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MATH CONCEPT LEARNING BY SPANISH SPEAKING CHILDREN: PROGRESS REPORT #1.

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400-77-0074

Submitted on: November 29, 1977.

Submitted by:

Dr. Jack A. Coffland

Dr. Gilberto J. Cuevas

School of Education

University of Miami

Coral Gables, Florida

33124

SE 023 986

PROGRESS REPORT #1

INTRODUCTION:

The basic scope of the study is to investigate the learning of mathematical concepts by primary grade students whose native language is other than English (Spanish). The efforts of the investigators in the study and the analysis of students' behaviors will be concentrated in the following areas:

a. The concept of number: This relates to the Piagetian concept of number, which includes amount, the ability to conserve amount over physical changes, the concept of reversability, and other such concepts discussed by Piaget.

b. The concept of numeration: Emphasis here is upon the use of a symbol substitution scheme as a way of teaching the child place value systems and the rules for organizing such systems. The child's learning of all numeration concepts will be traced developmentally.

c. The operations of addition and subtraction: The use of manipulatives and rule governed behavior will be studied as they relate to the teaching/learning process for basic operations.

d. The language involved in mathematics concepts: Second language patterns used to describe mathematics concepts will be examined. It is assumed that if the child does not understand the terms which are being used to describe a concept, he/she will not be able to understand the concept itself.

In order to obtain the necessary data in the above areas, the following research objectives will be accomplished.

a. Content analysis: A survey will be made of the existing Dade County Systems Math Curriculum to determine what aspects of number and numeration are emphasized. Other approved systems programs (approved for use in the State of Florida) will be examined for their content in number, numeration and the algorithms of addition and subtraction. A comprehensive list of objectives in these areas will be made so that children's learning of these topics can be examined and recorded.

b. Teacher survey: In order to determine the teachers' perceptions of which concepts are easy for the child to learn and which ones are difficult, a survey of teachers of primary age children will be undertaken. A list of mathematics objectives relating to number, numeration, addition and subtraction will be given; teachers will then be asked to indicate the perceived difficulty level for each of the objectives.

c. Classroom visitations: A third and major portion of the research procedures includes visitation in the classrooms selected. A case study approach of the students involved will be followed for the purpose of making developmental inferences. Data will be collected through:

1. observation/tutoring of children working on assigned objectives;

2. interviews of students in order to describe thinking patterns used when solving problems or completing algorithms;
3. video-tape of children's work with manipulatives;
4. audio-tape of responses to questions;
5. observation of teacher behavior and the language used in teaching math concepts;
6. testing and recording the children's use of math vocabulary and phrases.

STATUS OF ACTIVITIES AS RELATED TO RESEARCH OBJECTIVES:

A. Selection of Sample:

The protocol for the study was presented to the Dade County Research Review Committee and permission was granted to use Dade County students in the study, provided permission was obtained from parents giving the researchers access to student records. The protocol was also submitted to the University of Miami Committee on the Protection of the Rights of Human Subjects in Research Experiments. Again, approval was granted to do the study if parents signed an informed consent letter giving permission for their children to take part in the study. Students at West Laboratory Elementary School, however, sign a release to participate in research studies as they enter the school; this letter was deemed acceptable as the informed consent for West Lab students.

For students at West Lab, then, consent forms are on file in the principal's office. For students at Silver Bluff Elementary

School, signed consent forms and signed forms for the release of student records are on file in the office of the principal investigator. All students presently participating in the study have signed the school release form for West Laboratory Elementary School or have completed the informed consent letter and the release of records form if they attend Silver Bluff Elementary school. Examples of the informed consent letter and the release of records form are included in the Appendix. All forms were written in both English and Spanish.

As indicated in the research proposal, Silver Bluff Elementary School was selected as the school having a predominant number of Spanish language dominant children; West Laboratory Elementary School was chosen for English language dominant children. A first grade classroom in each school was selected for the purposes of the study. The following is a summary description of the participating student sample at each of the schools.

See Table I.

TABLE I

Description of Sample

By School, Sex, Age and Dominant Language

SCHOOL/LANGUAGE	AGE GROUP							
	5.5 - 5.9		6.0 - 6.4		6.5 - 6.9		7.0 +	
	Male	Female	Male	Female	Male	Female	Male	Female
SILVER BLUFF (n = 25)								
Spanish (n = 22)	-	-	6(24)	4(16)	8(32)	3(12)	1(4)	-
English (n = 3)	-	-	-	2(8)	-	1(4)	-	-
WEST LABORATORY (n = 26)								
Spanish (n = 1)	-	-	-	-	-	1(3.85)	-	-
English (n = 25)	-	1(3.85)	2(7.69)	12(46.15)	5(19.23)	5(19.23)	-	-

* Percentages are given in ( ).

As can be seen in Table I, the breakdown of students by age is approximately the same for both schools. All but two students are six years of age, as is the norm for first grade. The breakdown by sex indicates that the two school populations are decidedly different. Silver Bluff Elementary School has a predominantly male population, while West Lab Elementary School has a predominantly female population. Finally, it can be seen that Silver Bluff has a predominantly Spanish-speaking population, with only three students whose dominant language is English. West Lab, on the other hand, has a predominantly English-speaking population, with only one student whose dominant language is Spanish. West Lab also contains a sizeable number of Black students whose socio-economic status is decidedly lower than the white students in the classroom.

The teachers at both schools are female, with the instructor at Silver Bluff Elementary School having more years of experience than the one at West Laboratory School. In both classrooms children are grouped by ability for language arts and mathematics. The classroom at Silver Bluff is a low-average unit; the classroom at West Laboratory School is an average unit. Both teachers are competent professionals and were highly rated in professional activities by their principals.

B. Preliminary Data:

In order to establish a point of reference from which to proceed in the interpretation of the data obtained in the classroom visitations through the activities outlined on page 2, the following information has

been collected:

1. Dade County Systems Math Program:

Initial testing in the Dade County Systems Management program was done at the beginning of the school year in both classrooms being utilized in this study. Results of this testing have been recorded for each child; all future growth will be recorded so that the development of the child's knowledge in number, numeration, and the basic algorithms can be traced.

In the initial testing all children were operating somewhere in levels one through four. Levels one and two are normally assigned to the kindergarten year; levels three, four and five are normally assigned to the first grade year. All children, then, are either working on or slightly below grade level in the areas being studied.

Charts will be developed by the researchers tracing the record for each child in the objectives identified as pertaining to the areas of number, numeration, addition, subtraction, or other basic algorithms which might be introduced during the primary years. The identification of these objectives is part of the content analysis being made of existing systems management programs.

2. Metropolitan Readiness Test:

The MRT provides information about the pupil's levels of development in pre-reading and pre-mathematics skills. The results may be used by the teacher in planning instruction in reading and mathematics.

As stated in Teacher's Manual of the 1976 edition of the MRT, the MRT (Level II, Form P), is designed to assess the range of pre-reading and mathematical skills present in the child at the end of kindergarten and at the beginning of Grade I. The test consists of the following scales:

Auditory Memory:	This area assesses the child's immediate recall of initial consonant sounds and of sound/letter correspondence.
Visual Discrimination:	This area assesses the skills of visual matching and finding patterns.
Language Skills:	This area encompasses school language and listening skills, using standard American English.
Quantitative Skills:	This area covers quantitative concepts such as number-numeral correspondence, conservation of amount, part-whole relationships and quantitative operations, such as counting, addition and subtraction.

Data obtained will be utilized by the researchers to provide a context in which to relate the successes and/or failures experienced by pupils as they attempt to master the concepts of number, numeration and the basic algorithms. The results of the MRT are indicated in Table II.

See Table II.

TABLE II

Mean Scores On The

Metropolitan Readiness Test

LANGUAGE/ AGE GROUP	SUBSCALES							
	Auditory		Visual		Language		Quantitative	
	Male	Female	Male	Female	Male	Female	Male	Female
SPANISH								
5.5 - 5.9	-	-	-	-	-	-	-	-
6.0 - 6.4 yrs.	$\bar{x}$ = 19.67 SD = 5.20	$\bar{x}$ = 21.00 SD = 6.16	$\bar{x}$ = 17.33 SD = 5.75	$\bar{x}$ = 16.00 SD = 5.60	$\bar{x}$ = 10.83 SD = 2.64	$\bar{x}$ = 12.25 SD = .50	$\bar{x}$ = 15.00 SD = 4.69	$\bar{x}$ = 15.50 SD = 3.42
6.5 - 6.9 yrs.	$\bar{x}$ = 22.38 SD = 4.22	$\bar{x}$ = 25.00 SD = 2.45	$\bar{x}$ = 20.00 SD = 3.12	$\bar{x}$ = 22.00 SD = 4.08	$\bar{x}$ = 11.38 SD = 1.77	$\bar{x}$ = 10.75 SD = 2.87	$\bar{x}$ = 17.37 SD = 3.46	$\bar{x}$ = 16.25 SD = 3.10
7.0 -above yrs.	$\bar{x}$ = 27.00 SD = .00	-	$\bar{x}$ = 26.00 SD = 00	-	$\bar{x}$ = 8.00 SD = .00	-	-	-
ENGLISH								
5.5 - 5.9 yrs.	-	$\bar{x}$ = 29.00 SD = .00	-	$\bar{x}$ = 25.00 SD = .00	-	$\bar{x}$ = 15.00 SD = .00	-	$\bar{x}$ = 24.00 SD = .00
6.0 - 6.4 yrs.	$\bar{x}$ = 21.00 SD = 4.24	$\bar{x}$ = 26.50 SD = 2.93	$\bar{x}$ = 19.50 SD = .71	$\bar{x}$ = 20.86 SD = 3.06	$\bar{x}$ = 13.00 SD = 1.41	$\bar{x}$ = 13.07 SD = 2.95	$\bar{x}$ = 18.00 SD = 2.83	$\bar{x}$ = 17.64 SD = 2.32
6.5 - 6.9 yrs.	$\bar{x}$ = 28.60 SD = .55	$\bar{x}$ = 26.67 SD = 1.51	$\bar{x}$ = 23.20 SD = 2.49	$\bar{x}$ = 22.67 SD = 2.07	$\bar{x}$ = 14.60 SD = 1.67	$\bar{x}$ = 16.33 SD = .82	$\bar{x}$ = 20.40 SD = 2.07	$\bar{x}$ = 21.00 SD = 3.63
7.0 - above yrs.	-	-	-	-	-	-	-	-

The results of this test indicate a marked difference in performance between the two schools. (At this time, no statistical tests of significant differences have been made). As indicated in Table II, West Lab students are performing at a higher level for all subscales of this test and for all ages of the students. Due to the distinct differences between the schools, the assumption is made that school samples are derived from different populations. (Statistical verification will be made of this assumption). Therefore, no attempts will be made to compare the two groups statistically.

3. Administration of Children's Embedded Figures Test (CEFT):

Cognitive style was identified in the research proposal as a variable which may influence achievement in mathematics. The CEFT is a perceptual test; and as stated in the test manual: "The subject's task on each trial is to locate a previously seen simple figure within a larger complex figure which has been so organized as to obscure or embed the sought after simple figure". High ability on the test (norm = 6.8 for 5 - 6 year old males, SD = 3.8; 7.4 for 5 - 6 year old females, SD = 4.2) classifies a student as being field independent; low ability on the test classifies a student as being field dependent. For the purposes of this study, students will be classified on a continuum to locate students who are either field dependent or independent. An attempt will be made to gear instructional approaches to the cognitive style of the child. The CEFT was given to all of the students involved in the study late in October. The results were as follows:

See Table III.

TABLE III

Mean Scores on the

Children's Embedded Figures

Test

LANGUAGE GROUP	AGE GROUP							
	5.5 - 5.9 yrs.		6.0 - 6.4 yrs.		6.5 - 6.9 yrs.		7.0 + yrs.	
	Male	Female	Male	Female	Male	Female	Male	Female
Spanish	-	-	$\bar{x}$ = 8.67 SD = 1.51 n = 6	$\bar{x}$ = 6.25 SD = 2.22 n = 4	$\bar{x}$ = 8.00 SD = 2.83 n = 8	$\bar{x}$ = 6.00 SD = 3.16 n = 4	$\bar{x}$ = 11.00 SD = .00 n = 1	-
English	- - -	$\bar{x}$ = 8.00 SD = .00 n = 1	$\bar{x}$ = 9.00 SD = 1.41 n = 2	$\bar{x}$ = 9.57 SD = 3.27 n = 14	$\bar{x}$ = 12.40 SD = 3.65 n = 5	$\bar{x}$ = 12.33 SD = 4.55 n = 6	- - -	- - -

As can be seen in Table III, English-speaking children tended towards higher scores, indicating field independence. Cursary observation of the scores indicate that the Black students in West Lab Elementary School obtained the lower scores of the distribution. In the results obtained by Spanish-speaking children, scores were generally lower, with boys performing slightly higher than girls.

C. Content Analysis

The survey of the current Dade County Systems Math curriculum was completed. Appendix B contains a detailed list of the objectives, as presented in the instructional system and classified by cognitive area: number (both cardinal and ordinal concepts of number), numeration, and the basic algorithms of addition, subtraction, multiplication and division. The latter two operations were included as beginning objectives for these operations are presented in the primary grades and some children will be working on these objectives before the two year period of this study is completed.

After an initial classification of the objectives was made by the principal investigators, Dr. Pam Ferguson from the Mathematics Department reviewed the classification to cross-validate the placement of objectives under the different categories. Minor revisions will be made as a result of Dr. Ferguson's work.

It should be pointed out that the major suggestion presented by Dr. Ferguson is that many of the objectives presented in the objectives dealing with addition, subtraction, multiplication and division are in reality objectives dealing with some aspect of numeration. This bears out

the idea presented originally, that knowledge of numeration and place value is vitally important to the child's learning of arithmetic concepts. The following table includes those objectives which Dr. Ferguson felt could also be placed under the heading of numeration:

See Table IV.

TABLE IV

Objectives Dealing With Basic Algorithms

Which Also Deal With Numeration

Level	Oper.	Number	Objective
VII	Adding	11-22-01-01	Matches examples of commutative property to number lines.
		11-22-01-04	Complete addition equations, associative property, using number lines.
		11-22-01-06	Applies associative property (sums to 18.)
I	Sub.	11-31-01-01	Separates sets, identifies cardinal number of subsets.
VII		11-32-01-01	Subtraction as a non-commutative operation, number lines.
XI		11-32-01-03	Relates zero concepts to subtraction problems.
VII	Mult.	11-41-01-01	Interprets multiplication; identifies factor sets, product set, and completes equations for products less than or equal to 25.
IX		11-42-01-01	Identifies the multiplication properties of 0 and 1 by writing products or completing generalizations concerning the effect 1 and 0 have upon products.
		11-42-01-02	Using commutative and associative properties to complete multiplication number sentence and matrix.
XI		11-42-01-01	Multiplication properties of 1 and 0 (products 26 - 45.)
X	Div.	11-52-01-01	Division properties of 0 and 1.

The basic concept presented is that objectives dealing with the properties of our number system are also dealing with numeration. To this we would like to add one additional concept: All objectives dealing with regrouping in addition, subtraction or multiplication also deal with a proper understanding of the numeration system and should be included in both the objectives dealing with numeration and with those covering the basic operations.

In addition to the analysis of the Dade County Math Systems, two commercially developed programs have been requested in order to examine the objectives presented in these programs for the concepts of number, numeration, and the basic operations. As we have just received the programs, we have not yet had time to complete the analyses. The programs to be considered are:

The Silver Burdette Company's System Management Program.

The Ginn Company's Individualized Mathematics System.

By examining commercially developed programs we hope to present an analysis of objectives which would be more national in its scope.

In an effort to disseminate the information compiled, an article was submitted to a magazine devoted to the instructional practices of primary grade teachers, the Instructor magazine. A copy of that article has been included in Appendix D.

D. Teacher Survey:

A pilot test form of the Mathematics Objectives Teacher Survey (MOTS) has been developed and the objectives have been categorized by

instructional levels of the Dade County Systems Program. Such a categorization represents the actual teaching sequence of the objectives; teachers can then respond to the actual objectives they use for classroom instruction. Teachers will be asked to respond by rating each objective in terms of the perceived difficulty level of that objective for children. Appendix C contains a copy of the proposed inventory.

A random sample of schools using the systems math programs will be selected for use in the survey. Teachers in the first five grades (in this case, grades K, 1, 2, 3, 4) will be asked to complete the survey for those objectives centered in the grade levels they teach. In this case, that means that teachers in the following grades will respond to the objectives coming from the following levels:

Grade: K = Levels 1 and 2
1 = Levels 3, 4, and 5
2 = Levels 6, 7, and 8
3 = Levels 9, 10, and 11
4 = Levels 12, 13, and 14

This will give a sample of teachers who respond to the objectives that are taught most often in the grade level of the teacher. They should, therefore, be the individuals most knowledgeable about those particular objectives.

In order to make the questionnaire easy for teachers to respond to, the order of presentation of objectives follows the child's prescription

booklets. Teachers will then be able to use those prescriptions for reference when completing the questionnaire, making certain that all items are understood by all individuals completing the questionnaire. When the survey has been completed, an article describing the results will be submitted to the Arithmetic Teacher.

E. Classroom Visitations:

Classroom visitations have been conducted in an informal manner since approval as given by the Dade County Schools to conduct the study. The researchers have used the visitations to establish rapport with the students and teachers; to administer the identified standardized instruments given for beginning or baseline data, and to collect the necessary diagnostic data. No formal record of students' behaviors has been made to this point. Formal visitations are scheduled to begin immediately after the Thanksgiving holidays.

CONCLUSION

In conclusion, Table V presents a summary of the tasks accomplished to date. Table VI shows a projection of the activities which are expected to be completed during the period of time between the 3 and 6 month reports.

TABLE V
SUMMARY OF ACTIVITIES COMPLETED
(9.1.77 - 11.29.77)

Task	Completion Date	Person Responsible	Product
1.0 Research approval procedures			
1.1 - Dade County approval	8/31/77	Dr. J. Coffland	See letter Appendix A.
1.2 - University of Miami Research Council approval	9/15/77	Dr. J. Coffland	See letter Appendix A.
1.3 - Parental approval/ release	9/30/77	Dr. Coffland/Dr. Cuevas Ms. Ceccio/Paloma Ms. Frost/Fraga	See forms Appendix A.
1.4 - Teacher/Principal participation	9/15/77	Dr. Coffland/Dr. Cuevas Ms. Ceccio/Paloma Ms. Frost/Fraga	See letter Appendix A.
2.0 Selection of Participating schools	9/15/77	Dr. Coffland/Dr. Cuevas Ms. Frost/Ms. Fraga	See p.
3.0 Collection of base-line data			
3.1 - Diagnostic data/DCSM	9/77	Dr. Coffland/Dr. Cuevas Ms. Ceccio/Ms. Paloma	Material available on request.
3.2 - Children's Embedded Figures Test	10/21/77	Dr. Coffland/Dr. Cuevas Ms. Ceccio/Ms. Paloma	See Table III, p.
3.3 - Metropolitan Readiness Test	11/11/77	Dr. Coffland/Dr. Cuevas Ms. Ceccio/Ms. Paloma	See Table II, p.
3.4 - Review of relevant research/literature - ERIC Search	10/14/77	Dr. Coffland/Dr. Cuevas	ERIC listing available on request.
3.5 - Descriptive analysis of data for 3.1 - 3.4 above	11/23/77	Dr. Coffland/Dr. Cuevas	See Tables I, II, III.

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TABLE V Continued

Task	Completion Date	Person Responsible	Product
4.0 Content Analysis			
4.1 - Purchase of Dade County Math Systems	9/12/77	Dr. Coffland	Materials available on request.
4.2 - Orientation to Dade County Math Systems	9/12/77	Dr. Coffland/Dr. Cuevas Ms. Neff	None
4.3 - Purchase of a) IMS (Ginn and Co. b) Silver Burdette	Ord. 9/77 Rec. 11/77	Dr. Coffland/Dr. Cuevas	Materials available on request.
4.4 - Initial classification of objectives (Dade County)	9/30/77	Dr. Cuevas	n/a
4.5 - Review of classification	10/15/77	Dr. Coffland	n/a
4.6 - Cross validation review	11/11/77	Dr. Pam Ferguson	n/a
4.7 - Final classification list	In progress	Dr. Coffland/Dr. Cuevas	See Appendix B.
5.0 Teacher Survey			
5.1 - Development of pilot survey form/final form	11/18/77	Dr. Coffland	See Appendix C.
5.2 - Pilot test	12/77	Dr. Coffland/Dr. Cuevas	n/a
5.3 - Selection/description of sample	12/77	Dr. Coffland/Dr. Cuevas	n/a
5.4 - Administration of Math Objectives Teacher Survey	1/78	Dr. Coffland/Dr. Cuevas	n/a
5.5 - Analysis of results	2/78	Dr. Coffland/Dr. Cuevas	Not yet available.
6.0 Classroom visitation	On-going	Dr. Coffland/Dr. Cuevas Dr. Neumann	n/a

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TABLE VI
SUMMARY OF PROJECTED ACTIVITIES*
(12.1.77 - 2.29.78)

Task	Person Responsible	Product
1.0 Math Language Diagnostic Test	Coffland, Cuevas, Hudson, Ferguson	Math Language Diagnostic Test (MLDT)
1.1 Development of pilot test form (Spanish/English)		
1.2 Pilot test		
1.3 Revision		
1.4 Administration		
1.5 Analysis of results		
1.6 Validation procedures		
2.0 Classroom visitations	Coffland, Cuevas, Neumann, Ceccio, Paloma	
2.1 Observation (record of students' performance on assigned tasks		
2.2 Interview of students		
2.3 Tutoring		
2.4 Video-taping of students' activities with manipulatives		
2.4.1 Precise thinking exercises	Cuevas	
2.4.2 Symbol substitution exercises	Neumann	
2.5 Audio-taping of students' response to questions	Coffland, Cuevas, Neumann	
2.6 Observation of teacher behavior		

*Note: As a result of the exploratory nature of the research study, the list of activities should not be interpreted to be a final one. As data are collected, other research activities not identified here will be carried out.

APPENDIX A

Contents:

1. Authorization letter from Dade County to conduct the study.
2. Letter certifying review of the University of Miami Committee for the Protection of Human Subjects in Experiments.
3. Letter of consent for teachers.
4. Letter of consent for students:
    a. English version;
    b. Spanish version.
5. Release form for student records:
    a. English version;
    b. Spanish version.

DADE COUNTY PUBLIC SCHOOLS

ADMINISTRATIVE OFFICE
LINDSEY HOPKINS BUILDING
1410 N.E. 2ND AVENUE MIAMI, FLORIDA 33132

DR. J. L. JONES
SUPERINTENDENT OF SCHOOLS

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August 31, 1977

Dr. Jack A. Coffland
13240 S.W. 99 Court
Miami, Fla. 33176

Dear Dr. Coffland:

The Educational Research Committee has approved your request to conduct the following research project within the Dade County Schools:

AN INVESTIGATION INTO THE LEARNING OF MATH CONCEPTS BY STUDENTS WHOSE NATIVE LANGUAGE IS OTHER THAN ENGLISH (SPANISH).

Approval to conduct such a research project is subject to the following conditions:

1. Written parental approval to participate must be secured for all subjects. Parental approval forms should be prepared so that some basic information concerning the intent of the study and the nature of involvement, including time requirements, of the subjects are communicated. The availability of completed parental approval forms for each subject must be verified by the school principal or his/her designee.

2. Written parental approval must also be secured for access to any pupil's cumulative folder. The Dade County Schools' form Permission for Release of Records and/or Information from Records must be utilized for this purpose. These forms are available at any Dade County school. The procedures for securing such a release are defined in the publication Student Education Records (The School System: Office of the Deputy Superintendent) and must be conducted under the supervision of the principal or his/her designee.

3. Confidentiality of information for individual subjects must be maintained, individual-identifying information must be destroyed subsequent to the completion of the study, and information must not be disclosed to a third party.

As with all Educational Research Committee actions, approval of your research study does not constitute endorsement by the school system. In addition, the decision concerning whether an individual school will participate or not is totally at the discretion of the school principal.

I wish you every success in your research endeavor.

Sincerely yours,

Richard H. Hinds, Ed. D. Chairman
Educational Research Committee

RHH:rb

Principal Investigator

DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE

PROTECTION OF HUMAN SUBJECTS ASSURANCE/CERTIFICATION/DECLARATION

☒ ORIGINAL ☐ FOLLOWUP ☐ REVISION

☒ GRANT ☐ CONTRACT ☐ FELLOW ☐ OTHER

☒ NEW ☐ RENEWAL ☐ CONTINUATION

APPLICATION IDENTIFICATION NUMBER *(If known)*

4R01 HD-08-14

STATEMENT OF POLICY: Safeguarding the rights and welfare of subjects at risk in activities supported under grants and contracts from DHEW is primarily the responsibility of the institution which receives or is accountable to DHEW for the funds awarded for the support of the activity. In order to provide for the adequate discharge of this institutional responsibility, it is the policy of DHEW that no activity involving human subjects to be supported by DHEW grants or contracts shall be undertaken unless the Institutional Review Board has reviewed and approved such activity, and the institution has submitted to DHEW a certification of such review and approval, in accordance with the requirements of Public Law 93-348, as implemented by Part 46 of Title 45 of the Code of Federal Regulations, as amended, (45 CFR 46). Administration of the DHEW policy and regulation is the responsibility of the Office for Protection from Research Risks, National Institutes of Health, Bethesda, Md 20014.

1. TITLE OF PROPOSAL OR ACTIVITY

An Investigation into Children's Learning of Number and Numeration Concepts

2. PRINCIPAL INVESTIGATOR ACTIVITY DIRECTOR/FELLOW

Dr. Jack Coffland

3. DECLARATION THAT HUMAN SUBJECTS EITHER WOULD OR WOULD NOT BE INVOLVED

☐ A. NO INDIVIDUALS WHO MIGHT BE CONSIDERED HUMAN SUBJECTS, INCLUDING THOSE FROM WHOM ORGANS, TISSUES, FLUIDS, OR OTHER MATERIALS WOULD BE DERIVED, OR WHO COULD BE IDENTIFIED BY PERSONAL DATA, WOULD BE INVOLVED IN THE PROPOSED ACTIVITY. (IF NO HUMAN SUBJECTS WOULD BE INVOLVED, CHECK THIS BOX AND PROCEED TO ITEM 7. PROPOSALS DETERMINED BY THE AGENCY TO INVOLVE HUMAN SUBJECTS WILL BE RETURNED.)

☒ B. HUMAN SUBJECTS WOULD BE INVOLVED IN THE PROPOSED ACTIVITY AS EITHER: ☐ NONE OF THE FOLLOWING, OR INCLUDING: ☒ MINORS, ☐ FETUSES, ☐ ABORTUSES, ☐ PREGNANT WOMEN, ☐ PRISONERS, ☐ MENTALLY RETARDED, ☐ MENTALLY DISABLED. UNDER SECTION 6. COOPERATING INSTITUTIONS, ON REVERSE OF THIS FORM, GIVE NAME OF INSTITUTION AND NAME AND ADDRESS OF OFFICIAL(S) AUTHORIZING ACCESS TO ANY SUBJECTS IN FACILITIES NOT UNDER DIRECT CONTROL OF THE APPLICANT OR OFFERING INSTITUTION.

4. DECLARATION OF ASSURANCE STATUS/CERTIFICATION OF REVIEW

☐ A. THIS INSTITUTION HAS NOT PREVIOUSLY FILED AN ASSURANCE AND ASSURANCE IMPLEMENTING PROCEDURES FOR THE PROTECTION OF HUMAN SUBJECTS WITH THE DHEW THAT APPLIES TO THIS APPLICATION OR ACTIVITY. ASSURANCE IS HEREBY GIVEN THAT THIS INSTITUTION WILL COMPLY WITH REQUIREMENTS OF *DHEW Regulation 45 CFR 46*, THAT IT HAS ESTABLISHED AN INSTITUTIONAL REVIEW BOARD FOR THE PROTECTION OF HUMAN SUBJECTS AND, WHEN REQUESTED, WILL SUBMIT TO DHEW DOCUMENTATION AND CERTIFICATION OF SUCH REVIEWS AND PROCEDURES AS MAY BE REQUIRED FOR IMPLEMENTATION OF THIS ASSURANCE FOR THE PROPOSED PROJECT OR ACTIVITY.

☒ B. THIS INSTITUTION HAS AN APPROVED GENERAL ASSURANCE (DHEW ASSURANCE NUMBER G0301) OR AN ACTIVE SPECIAL ASSURANCE FOR THIS ONGOING ACTIVITY, ON FILE WITH DHEW. THE SIGNER CERTIFIES THAT ALL ACTIVITIES IN THIS APPLICATION PROPOSING TO INVOLVE HUMAN SUBJECTS HAVE BEEN REVIEWED AND APPROVED BY THIS INSTITUTION'S INSTITUTIONAL REVIEW BOARD IN A CONVENED MEETING ON THE DATE OF 8-18-77 IN ACCORDANCE WITH THE REQUIREMENTS OF THE *Code of Federal Regulations on Protection of Human Subjects (45 CFR 46)*. THIS CERTIFICATION INCLUDES, WHEN APPLICABLE, REQUIREMENTS FOR CERTIFYING FDA STATUS FOR EACH INVESTIGATIONAL NEW DRUG TO BE USED (SEE REVERSE SIDE OF THIS FORM).

THE INSTITUTIONAL REVIEW BOARD HAS DETERMINED, AND THE INSTITUTIONAL OFFICIAL SIGNING BELOW CONCUR THAT:

EITHER ☐ HUMAN SUBJECTS WILL NOT BE AT RISK; OR ☒ HUMAN SUBJECTS WILL BE AT RISK.

5. AND 6. SEE REVERSE SIDE

7. NAME AND ADDRESS OF INSTITUTION

University of Miami
Coral Gables, Florida

8. TITLE OF INSTITUTIONAL OFFICIAL	TELEPHONE NUMBER
Mr. Michael J. Moloney	(305) 284-6610
SIGNATURE OF INSTITUTIONAL OFFICIAL	DATE
Michael J. Moloney	9/15/77

HEW-596 (Rev. 4-75)

ENCLOSE THIS FORM WITH THE PROPOSAL OR RETURN IT TO REQUESTING AGENCY.

INFORMED CONSENT LETTER

You are being asked to participate in a research study in the area of math for the coming year. The purpose of the research study is to investigate the way in which young children learn arithmetic concepts.

As a part of this study University of Miami professors will be observing in your classroom. While their primary task is to work with the children in your classroom, at times your presentation of math concepts may be recorded in order to analyze the language used in teaching math. These tapes will be stored for two years and then destroyed; the information on them will not be released without your approval.

If you give your permission to take part in this study, you may withdraw your consent at any time.

I hereby give my informed consent to allow this research study to be done in my classroom.

_____ _____

Date Teacher's Signature

INFORMED CONSENT LETTER

Your child is being asked to participate in a research study in the area of math for the coming two years. The purpose of the research study is to investigate the way in which young children learn arithmetic concepts.

As a part of this study your child may be asked to take part in a series of interviews with a University of Miami professor, he may be tutored in some math concept by a University professor, or he may be observed in his work by a University professor. He may also be given diagnostic achievement tests in the area of math. Your child's responses may also be tape recorded. These tapes will be stored for 2 years and then destroyed; the information on them will not be released without your approval.

Also as a part of this study the achievement records of your child will be made available to the research staff. All the information will be held in the strictest confidence; no data will be released in such a way that your child could be identified.

If you give your permission for your child to take part in this study, you may withdraw the consent at any time. Your child will then drop out of the study.

I hereby give my informed consent to allow my child to participate in this research study.

_____ _____

Date Child's Name

_____

Parent's Signature

CARTA DE AUTORIZACIÓN

Su hijo/hija a sido escogido como participante en un estudio en el area de matemáticas por los próximos dos años. El estudio investigará la forma en que los niños aprenden conceptos aritméticos.

Como parte de este estudio, se le pedirá a su hijo/hija que participe en una serie de entrevistas con un profesor de la Universidad de Miami. El profesor también asistirá al alumno en el aprendizaje de aritmética por medio de clases, observaciones y exámenes diagnósticos.

Sus respuestas en los tests serán grabadas, estas grabaciones serán archivadas por dos años. Después de estos dos años, las grabaciones serán destruidas. La información en dichas grabaciones no sera publicada o usada en ninguna forma sin el consentimiento de los padres.

Si Ud. consiente que su hijo/hija forme parte de este estudio llene y firme los espacios correspondiente. Usted puede retirar el permiso en cualquier momento, y su hijo/hija sera retirado del programa.

_____ _____

Fecha Nombre de su hijo/hija

_____

Firma del Padre

DADE COUNTY PUBLIC SCHOOLS

PERMISSION FOR RELEASE OF RECORDS
AND/OR INFORMATION FROM RECORDS

Student's Name: _____ DOB: _____

Records to be Released: (Please check appropriate item(s))

Psychological Report	x	
Scholastic Grades	x	
Test Scores	x	
Health/Medical Records		
Attendance Information	x	
Other (Specify)	x	Systems Math Achievement Record

The record(s) indicated above is/are to be released to:

Agency University of Miami

Address School of Education, Coral Gables, Florida 33124

Contact Person(s) Dr. Jack A. Coffland or Dr. Gilberto J. Cuevas

The purpose for this release is: Provide data for study entitled: An Investigation into Children's Learning of Number and Numeration Concepts.

This release is to be in effect until 8/31/79 (Date)

I hereby grant permission for the release of the above record(s)

_____ _____
Signature of Parent or Guardian (Date)

A certified copy of this authorization shall be as valid as the original.

Silver Bluff Elementary School
Name of School/Agency Releasing/Requesting Record(s)

_____
Signature of Authorized Personnel

_____
Title

The Dade County Public School System is subject to the Family Rights and Privacy Act of 1974 (Public Law 93-380). Therefore, all documents contained in students' records are accessible to the parents for their review upon request.

MIS-11867 (04-77)

DADE COUNTY PUBLIC SCHOOLS

PERMISO PARA SUMINISTRAR DOCUMENTOS O
REVELAR INFORMACION PERTINENTE

Nombre del Alumno _____ Fecha de Nacimiento _____

Documentos Requeridos (Por favor indique):

Informe Psicológico	x
Resultados de Pruebas	x
Historia Médica	
Asistencia Escolar	x
Otro (Especifique)	x Sistemas Diagnostico de Matematica

El documento o documentos requeridos serán suministrados a:

Agencia(s) Universidad de Miami

Dirección(es) Escuela de Educación, Coral Gables, Florida

Personal Autorizado Dr. Jack A. Coffland o Dr. Gilberto Cuevas

Los documentos o información serán suministrados con el objeto de: Para proporcionar datos sobre los estudios titulados: Una Investigación sobre el aprendizaje de los numeros y conceptos numericos en los niños.

Esta autorización será válida hasta: 8/31/79 (Fecha)

Por este medio autorizo que los documentos requeridos o copias de los mismos sean suministrados.

_____ _____
Firma del padre, tutor o interesado (Fecha)

Una copia certificada de esta autorización será tan válida como la original.

Silver Bluff Elementary School
Escuela/Agencia que Suministra/Recibe los Documentos

_____
Firma del personal autorizado

_____
Título

El Sistema de Escuelas Públicas del Condado Dade se rige por los Derechos de Familia y Privacidad (Family Rights and Privacy Act, Public Law 93-380). Por lo tanto todos los documentos incluídos en el expediente del estudiante son accesibles a revisión por parte de los padres que así lo requieran.

MIS-11867 (04-77)

APPENDIX B

Contents: Objectives dealing with:

Page	Content
1	Cardinal principle of number
2	Ordinal principle of number
3	Numeration
6	Special numeral systems
8	Addition
12	Subtraction
16	Multiplication
19	Division

CONTENT AREA: NUMBER: Cardinal Principle of Number

Level	Number	Objective
LEVEL I	11-71-02-01	Identifies sets: more, fewer, same number of elements
	11-71-01-04	Orders sets, smallest number to largest number
	11-11-01-01 11-11-01-02	Identifies sets equal or unequal in number
	11-71-01-01	Identifies sets equal or unequal in number
	11-71-01-03	Makes second set equal in number to first set
	11-11-01-05	Makes non-equivalent sets equivalent
	11-11-03-01 11-11-03-04	Learner selects counters to show number of things, to ten
	11-11-03-01a 11-11-03-04a	Learner selects numeral cards to show number of things to ten
LEVEL II	11-71-02-01 11-71-02-02	Identifies set that has more or fewer members
	11-71-02-03	Identifies set that has fewest, least, greatest, most members
	11-11-01-03	Identifies set having one more or one less, 0 - 5
	11-11-01-04	Identifies set having one more or one less, 6 - 10
	11-11-01-06	Establishes set equivalence
	11-11-03-02	Matches set with cardinal number, 0 - 5
	11-11-03-03	Matches set with cardinal number, 6 - 10
LEVEL VI	11-71-03-02	Uses < or > to describe order relation between pairs.
	11-71-03-01	Uses = or ≠ in number sentences.
LEVEL VIII	11-71-02-07	Identifies larger or smaller number, 11- 100
	11-71-02-08	Identifies smallest or greatest number in a triplet of numbers, 11 - 100
	11-71-03-03	Uses = or ≠ to describe related numbers
	11-71-03-04	Uses < or > to relate numbers 11 - 100

CONTENT AREA: NUMBER: Ordinal Principle of Number

Level	Number	Objective
LEVEL I	11-11-06-02	Places picture in specific ordinal position
LEVEL III	11-11-06-01	Marks object in ordinal position, first to fifth
LEVEL V	11-11-06-03	Matches ordinals to object in positional order, first through fifth.
	11-11-07-01 11-11-07-02 11-11-07-03	Labels ordinal position of pictured events.
LEVEL VII	11-11-06-04	Matches ordinal words to given positions; 6th - 10th
LEVEL IX	11-11-06-05	Reads written ordinal words and matches to pictured ordered items in those positions, 11th to 20th
	11-11-06-06	Matches ordinal abbreviations to ordered pictured items, 1 - 19.
	11-11-06-07	Selects appropriate written ordinal word names for ordinal numbers, 25 - 100.

CONTENT AREA: NUMERATION

Level	Number	Objective
LEVEL I	11-31-01-03 11-11-03-01a 11-11-03-02a	Identifies cardinal number of two sets, comparison Learner selects numeral cards to show number of things to 10.
LEVEL II	11-71-02-04	Identifies numeral that names greater number, 10 or less.
	11-71-02-05	Identifies numeral that names smaller number.
	11-71-02-06	Identifies numeral that names the largest number.
LEVEL III	11-11-02-01 11-11-04-01	Writes standard numeral.
	11-11-02-02 11-11-04-02	Writes standard numeral.
	11-11-02-03 11-11-03-05	Matches numeral to collection.
	11-11-02-03 11-11-04-03	Writes standard numeral
	11-11-05-01	Writes standard numeral for written word names, 0 - 10
LEVEL IV	11-41-01-016	Identifies N sets of M number of objects, N is 12 or less.
LEVEL V	11-11-02-04 11-11-03-06 11-11-03-07	Writes standard numeral from pictured collections, to 50.
	11-11-09-02	Selects standard numeral from picture, to 99.
	11-11-08-01 11-11-08-04	Counts by two's to 18, by ten's to 50.
	11-11-09-01	Writes expanded numeral from pictures, ten and ones. (to 99.)
	11-11-10-01 11-11-11-01	Writes standard numeral for standard expanded form.
	11-11-12-01	Writes expanded form for standard form.

CONTENT AREA: NUMERATION

Level	Number	Objective
LEVEL VII	11-11-08-02 11-11-08-03	Counts by three's to 18; by five's to 20.
	11-11-08-05	Gives cardinal number of sets grouped by 50's, 100's, for sets 200 to 100.
	11-11-09-03	Supplies expanded numeral from pictures, 100 - 999.
	11-11-10-02	Interprets positional value of specified digits, 3 digits.
	11-11-11-02	Selects or writes standard form or other expanded form for three-digit numeral.
LEVEL VIII	11-71-02-01	Identifies larger or smaller numeral, n to 100.
	11-71-02-08	Identifies smallest or greatest numeral in a triplet, 11 - 100.
LEVEL IX	11-11-05-02	Writes standard numerals for written word names; 11 - 99.
	11-11-05-03 11-11-05-04	Writes standard numerals for written word names, 100 - 1000.
LEVEL XI	11-11-12-02	Writes standard numeral for pictures, 1000 - 3000.
	11-11-12-03	Writes expanded form, 100 - 999.
	11-11-05-06	Matches numeral to written word name, 100 - 1000.
LEVEL XII	41-32-01-01	Identifies odd and even numerals, 1 - 100.
LEVEL XIII	11-11-05-07 11-11-05-08	Supplies numeral for written number, over 1000.
	11-11-09-04 11-11-11-03	Writes standard numeral or expanded form for written form.
	11-11-12-04 11-11-02-05	Writes standard numeral or expanded form for $(a \times 10) + b$ or $(a \times 0) + b$ where a and b are less than 10.
	11-11-03-01	Rounds 2-digit numerals to nearer ten.
	41-32-01-01 41-32-01-02	Classifies sums as odd or even.

CONTENT AREA: NUMERATION

Level	Number	Objective
LEVEL XIII (con't)	41-32-01-03	Classifies products as odd and even.
	41-11-01-01	Classifies number to 100 as prime, composites, or neither.
	41-11-01-04	Labels statements about primes and composites as T or F.
LEVEL XV	11-11-10-04	Gives value for specified digit, 4 - 6 digits,
	11-11-11 04	Standard numeral given for expanded form, 4 - 6 digits.

CONTENT AREA: **NUMERATION (Special Systems)**

Level	Number	Objective
LEVEL I	31-21-01-01	Identifies penny, nickel, dime.
LEVEL IV	31-31-02-01 31-31-02-02	Draws hour on clock.
	31-31-02-03 31-31-02-04	Draws hour hand on clock for half-past.
LEVEL VI	31-21-01-02	Identifies quarters and half dollars.
	31-21-01-02	Learner marks coins given in cents.
	31-21-02-01	Finds different ways to make nickel; dime.
	31-21-02-02	Finds different ways to make quarter.
	31-31-02-05	Makes clock show __:00 o'clock.
	31-31-02-06	Makes clock show __:30 o'clock.
	31-31-03-01	Arranges in order: days of the week.
LEVEL VIII	31-21-01-04	Matches pictures of coins to their values in cent notation: ≤ 25¢.
	31-21-01-05	Matches pictures of coins to written words: values to 25¢.
	31-21-01-06	Matches pittures of coins to value in dollar notation: ≤ 25¢.
	31-21-01-07	Writes values of pictured coins in both cent and dollar notation: ≤ 25¢.
	31-31-03-02	Arranges written names of the month in order.
	31-31-03-03	Identifies time lapsed as ___ months ___ weeks ___ days.
LEVEL X	31-21-02-02	Identifies combined equivalent to quarter.
LEVEL XI	31-21-01-04	Matches coins to values in ¢ (≤ 50¢).
	31-21-01-05	Matches coins to written word value half-dollar.

CONTENT AREA: **NUMERATION (Special Systems)**

Level	Number	Objective
LEVEL XI (con't)	31-21-01-06	Matches coins to values; dollar notation (≤ 50).
	31-21-01-07	Writes value of coins to 50 cents; cent and dollar notation.
LEVEL XII	31-21-02-04	Purchases; fewest coins. Total ≤ $1.00.

CONTENT AREA: **ALGORITHMS (Addition)**

Level	Number	Objective
LEVEL I	11-21-01-01 11-21-01-02	Identifies cardinal number of 2 sets and cardinal number of union of 2 sets.
	11-41-01-01	Removes sets of specified cardinal number.
LEVEL III	11-21-01-04 11-21-02-01 11-21-02-03	Matches addition equation to pictured union sums to 10.
	11-21-02-09	Writes addition equation from number line, sums to 10.
	11-21-02-11	Writes equations to addition situations, sums to ten.
	11-23-01-01 11-23-01-02	Writes the sum of 2 addends, horizontal and vertical, sums to 10.
	11-23-01-05	Writes missing term (sum or addend), sums to 10.
	11-31-01-03 11-31-01-07 11-31-01-08	Translates number line representation into equations, sums to 10.
	11-33-01-04	Writes missing numerals (sums or addends), subtraction, sums to 10.
LEVEL IV	11-23-01-07	Horizontal form, sums to 10 one-minute timed test (20 problems).
	11-23-01-08	Vertical form, sums to 10 one-minute timed test (20 problems).
LEVEL V	11-21-01-03 11-21-01-05 11-21-02-04	Matches additional number sentences; pictorial representations used, sums 11 to 18.
	11-21-02-05	Writes number sentences; pictorial representations used, sums to 10.
	11-21-02-06	Writes number sentences; pictorial representations used, sums to 11 to 18.
	11-21-02-07	Writes number sentences from number line (sums to 11 to 18).
	11-21-02-10	Translates oral statements into an equation; sums to 18.

CONTENT AREA: **ALGORITHMS (Addition)**

Level	Number	Objective
LEVEL V (con't)	11-21-02-11	Identifies "plus," "addend," "sum," "equals," or "add."
	11-21-02-12	Groups addends by two's to find sums to 10.
	11-21-02-13	Groups addends by two's to find sums to 18.
	11-23-01-03	Finds sums (up to 4 addends) 1 digit addends (sums to 10).
	11-23-01-04	Completes addition matrix sums to 10.
	11-23-01-06	Finds missing terms (vertical form) (up to 4 addends) (sums to 10).
	11-23-02-01	Sums to 18; 2 addends horizontal form.
	11-23-02-02	Sums to 18; 2 addends vertical form.
LEVEL VI	11-21-02-14	Matches words to specific parts of addition equation (add, plus, equals, sums, addend).
	11-24-01-03 11-24-02-01	Matches addition word problem to equation (sums to 10).
	11-24-02-02 11-24-03-01 11-24-03-04 11-24-02-03	Writes addition equation for word problem to equation, sums to 10.
	11-24-01-02 11-24-01-04 11-24-03-02 11-24-03-03	Writes equation and solution for word problems, sums thru 18.
	11-24-03-04	Reads word problems and writes solution (sums to 10).
	11-24-03-05	Reads and writes solution sums to 18.
LEVEL VII	11-22-01-01	Matches examples of comm. prop. to number lines.
	11-22-01-02	Gives cardinal number of the union of sets as 10 plus ___, (sums 11 - 18).

CONTENT AREA: **ALGORITHMS (Addition)**

Level	Number	Objective
LEVEL VII (Con't)	11-22-01-03	Writes equations: form a + b = b + a using number lines.
	11-22-01-04 11-22-01-05	Complete addition equations, associative property, using number lines.
	11-22-01-06	Applies associative property (sums to 18).
	11-23-02-03	Completes addition matrices; sums 11 - 18.
	11-23-02-04	Addition equations; 2, 3, 4 addends missing terms; sums 11 - 18; horizontal.
	11-23-02-05	Addition equations; 2, 3, 4 addends missing terms; sums 11 - 18; vertical.
	11-23-02-06	One minute timed test; horizontal, sums 11 - 18.
	11-23-02-07	One minute timed test; vertical, sums 11 - 18.
	11-23-02-08	Vertical form; 3 - 4 - 5 digit addends sums to 18.
	11-23-03-01	Sums, 1 and 2 - digit addends; horizontal and vertical; no regrouping.
	11-23-03-02	Sums, 1 and 2 - digit addends; horizontal and vertical; regrouping.
	11-23-04-01	Sums, 2 or 3 digit addends, vertical; no regrouping.
LEVEL VIII	11-24-02-02	Writes equation for word problems, sums through 18.
	11-24-03-02	Solves word problems; sums 11 - 18.
	11-62-01-04	Selects addition equations from subtraction equations. Sums ≤ 10.
	11-62-01-06	Selects addition equations from subtraction equations; sums 11 - 18.
	31-23-02-01	Writes sums in cent notation: sums $\leq$ 10¢.
	31-23-02-02	Writes sums in cent notation: sums $\leq$ 20¢.
	31-24-01-02a	Writes total value of pictured items labeled as to value; value $\leq$ 10¢.
	31-24-01-02	Finds value of selected items and amount of change. Value $\leq$ 10¢.

CONTENT AREA: **ALGORITHMS (Addition)**

Level	Number	Objective
LEVEL IX	11-21-02-08	Writes addition equations from verbal descriptions of additive situations.
	11-31-01-04 11-31-01-05	Determines number by which one set has more or fewer items and selects appropriate sentence.
LEVEL X ✓	11-62-01-07	Matches addition equation to subtraction equation. Sums ≤ 100.
	31-24-01-01	Writes total value of labeled items. Value ≤ 25¢.
	31-24-01-03	Selects items; finds total value. Writes change. Values ≤ 25¢.
LEVEL XI	11-23-04-02	Sums, 2 or 3 digit addends, vertical; regrouping.
	11-24-01-05 11-24-01-06	Matches word problem to addition equation. Sums 100.
	11-24-02-03	Writes equation for addition word problems; sums 100.
	11-24-03-06 11-24-03-07	Writes solutions for addition word problems sums (100 - 1000).
	11-24-03-08	Reads and solves addition word problems, sums ≤ 1000.
LEVEL XII	31-23-01-04	Adds money, dollars and cents, horizontal and vertical.
LEVEL XIII	11-23-02-08 11-23-02-11	Missing terms, 2 - 3 digit addition regrouping.
LEVEL XV	11-23-04-03	Finds sums, vertical form, six or less digits.

CONTENT AREA: **ALGORITHMS (Subtraction)**

Level	Number	Objective
LEVEL I	11-31-01-01 11-31-01-02	Separates sets, identifies cardinal number of subsets.
LEVEL III	11-31-01-03 11-31-01-07 11-31-01-10	Matches number line to subtraction sentence (sums to 10).
	11-31-01-03 11-31-01-07 11-31-01-09	Matches picture to related subtraction sentence (take away).
	11-31-01-03 11-31-01-07 11-31-01-09	Matches picture to related subtraction sentence (comparative).
	11-33-01-01 11-33-01-02 11-33-01-03	Writes differences, vertical and horizontal form, sums to 10.
LEVEL IV	11-33-01-05	Horizontal forms, differences to 10, one-minute timed tests (20 problems).
	11-33-01-06	Vertical forms, differences to 10, one-minute timed tests (20 problems).
LEVEL V	11-31-02-01	Identifies "minus," "equal," "subtract," "minuend," "subtrahend," "difference," "sum," "addend."
	11-31-01-03b	Horizontal form, sums to 18, supplies differences.
	11-31-01-03b 11-33-02-02	Vertical form, sums to 18, supplies the differences.
	11-31-01-03b 11-32-02-04	Writes missing terms, horizontal and vertical forms - - sums 11 to 18.

CONTENT AREA: **ALGORITHMS (Subtraction)**

Level	Number	Objective
LEVEL VI	11-34-01-01 11-34-01-02 11-34-01-03 11-34-02-01	Translates problems into subtraction story problems. (Sums thru 10).
	11-34-01-04	Matches story problem to corresponding equations (sums thru 18).
	11-34-02-02 11-34-02-03	Matches subtraction word problem to equation (sums 11 to 18).
LEVEL VII	11-32-01-01	Subtraction as non-commutative operation number lines.
	11-33-02-05	Subtraction; one minute timed test horizontal sums 11 - 18.
	11-33-02-06	One minute timed test, subtraction; vertical, sums 11 - 18.
	11-33-03-01	Supplies differences 1 and 2 digit; vertical; no regrouping.
	11-33-03-02	Supplies differences 2 and 3 digit numerals; no regrouping.
	11-33-03-03	Supplies missing terms, subtraction; 1 and 2 digit; no regrouping.
	11-33-03-04	Supplies missing terms, subtraction; 2 and 3 digit; horizontal, no regrouping.
	11-33-03-05	Supplies missing terms, subtraction; up to 3 digits; norizontal and vertical; no regrouping.
LEVEL VIII	11-62-01-01 11-62-01-02	Selects subtraction equations from addition equations. Sums ≤ 10.
	11-62-01-05	Selects subtraction equations from addition equations. Sums 11 - 18.
	31-33-02-01	Writes differences in cent notation: sums $\leq 10¢$.
	31-33-02-02	Writes differences in cent notation: sums $\leq 20¢$.

CONTENT AREA: **ALGORITHMS (Subtraction)**

Level	Number	Objective
LEVEL IX	11-31-01-04 11-31-01-05	Determines number by which one set has more or fewer items and selects appropriate number sentences.
	11-31-01-06	Supplies or selects number sentences for pictured set subtraction scenes.
	11-34-01-05	Writes word problems for pictured subtraction situations.
	11-34-01-06 11-34-01-07 11-34-01-08	Selects subtraction story problems for $a - b = c$, $a \leq 18$.
	11-34-03-01	Reads written subtraction word problems with examiner and solves; sums to 10.
	11-34-03-02	Reads written subtraction word problems with examiner and solves; sums 11 - 18.
LEVEL X	11-62-01-07	Matches subtraction equation to addition equation. Sums ≤ 100.
	31-24-01-03	Selects items; finds total value. Writes change. Values $\leq 25¢$.
LEVEL XI	11-31-02-01 11-31-02-02	Selects best minuend, regrouping tens to ones; hundreds to tens; 2 and 3 digits.
	11-31-02-03 11-31-02-05	States whether regrouping is needed, 2 and 3 digit subtraction problems.
	11-32-01-03	Relates zero concepts to subtraction problems.
	11-34-02-03 11-34-02-04 11-34-02-05 11-34-02-07	Selects equations for subtraction word problems; 2 and 3 digits; no regrouping.
	11-34-02-06	Equations which are not translations of subtraction word problems, 2 and 3 digits, no regrouping.
	11-34-03-03	Solves subtraction word problem sums (18 - 99) no regrouping.

CONTENT AREA: **ALGORITHMS (Subtraction)**

Level	Number	Objective
LEVEL XII	31-23-01-03	Subtracts money cent notation, no regrouping, sums ≤ $1.00.
	31-23-02-05	Subtracts money, dollar and cents, no regrouping, sums ≤ $10.00.
LEVEL XIII	11-33-03-02 11-33-03-03 11-33-03-04	Missing terms, 2 - 3 digit subtraction.
	11-33-03-05	Finds differences, 2 - 3 digit subtraction, no regrouping.
	11-33-03-06 11-33-03-07	Finds differences, 1, 2, 3 digit subtraction. Regrouping.
	11-33-03-07 11-33-03-08	Missing terms, 2 - 3 digit subtraction. Regrouping.
	11-34-03-07	Subtraction word problems, sums 18 to 99, regrouping.
	31-23-02-04	Subtracts money, cent notation, to 99¢ with regrouping.
LEVEL XV	11-33-04-01 11-33-04-02	Finds differences (4 - 6 digit) horizontal and vertical without regrouping.
	11-33-04-03	Supplies missing terms, vertical subtraction without regrouping.
	11-33-04-04 11-33-04-05	Subtraction (4 - 6 digit) horizontal and vertical, with regrouping.
	11-33-04-06	Supplies missing terms (4 - 6 digit) horizontal with regrouping.

CONTENT AREA: ALGORITHMS (Multiplication)

Level	Number	Objective
LEVEL VIII	11-41-01-01	Interprets multiplication; identifies factor set, product set, and completes equations. Products ≤ 25.
	11-44-01-01 11-44-01-04	Translates word problems into equations and solves products ≤ 25.
LEVEL IX	11-41-01-02	Supplies expressions from pictured set situations; a x b, products ≤ 25.
	11-41-01-04	Interprets expressions, horizontal form, $a_1 + a_2 + a_3 + \ldots + a_n$ = n sets of a or n · a, product ≤ 25.
	11-41-01-05	Interprets expressions, vertical format $a_1 + a_2 + \ldots + a_n$ = n sets of a or n · a, products ≤ 25.
	11-41-01-06 11-41-01-07	Writes multiplication and addition number sentences; products and sums ≤ 25.
	11-41-01-08	Supplies number names of equal addends and related factors from number lines showing repeated addition.
	11-41-01-09 11-41-01-10	Supplies multiplication sentences from arrays.
	11-41-04-01	Uses multiplication to name the number of possible pairings between elements of two sets # of pairings ≤ 25.
	11-41-04-02	Organizes pairings of 2 sets to show multiplication situations ≤ 25.
	11-42-01-01	Identifies the multiplication properties of 1 and 0 by writing products or completing generalizations concerning the effect 1 and 0 have upon products.
	11-42-01-02 11-42-01-03	Uses commutative and associative properties to complete multiplication number sentence and matrix.
	11-43-01-01 11-43-01-02	Writes products in horizontal and vertical form, products to 25.
	11-43-01-03	Completes multiplication sentences and matrix products to 25.
	11-43-01-04	Two-minute timed test, products to 25, vertical form.

CONTENT AREA: ALGORITHMS (Multiplication)

Level	Number	Objective
LEVEL X	11-44-01-01 11-44-01-05	Multiplication word problems equations and solutions: products ≤ 25.
	11-62-02-03	Matches multiplication equations to division equations: products ≤ 25.
LEVEL XI	11-41-01-01	Multiplication facts from pictures; products 26 - 45.
	11-41-01-02 11-41-01-03	Product expressions from pictures 26 - 45.
	11-41-01-04/05/06/07	Addition and multiplication equations from number lines (26 - 45).
	11-41-01-09 11-41-01-10	Multiplication equations from rectangular arrays (26 - 45).
	11-41-05-01	Matches words to symbols in addition and multiplication equations (26 - 45).
	11-42-01-01	Multiplication properties of 1 and 0 (products 26 - 45).
	11-42-01-02 11-42-01-03 11-43-02-01	Complete multiplication matrix products to 45.
	11-43-02-02 11-43-02-03	Products through 25, horizontal and vertical.
	11-43-06-01	Matches number to its multiples to 45.
	11-44-01-02	Writes equation and solves multiplication word problems: products to 45.
LEVEL XII	11-43-02-04	Products; 1 minute timed test; products ≤ 45.
	31-43-04-01	Multiplies money by whole number; dollar notation, product ≤ 45¢.
	31-43-04-02	Multiplies money by whole number; dollar notation, product ≤ $.45.

CONTENT AREA: **ALGORITHMS (Multiplication)**

Level	Number	Objective
LEVEL XIII	11-41-01-01/02/03	Matches expressions to picture products.
	11-41-01-04	Matches products to repeated addition products 46 - 81.
	11-41-01-06 11-41-01-07	Writes addition and multiplication equations from picture products 46 - 81.
	11-41-01-08	Number lines, repeated addition and multiplication. Products 46 - 81.
	11-41-01-09 11-41-01-10	Writes multiplication equations from arrays. Products 46 - 81.
	11-43-01-05	Missing terms from multiplication facts to 81.
	11-43-03-01 11-43-03-02	Writes products to 81.
	11-43-06-03	Writes multiplication expressions for multiples of N (46 - 81).
	11-43-06-04 11-43-06-05	Names multiples of N; Identifies N from a set of multiples of N.
	11-44-01-03	Writes equations and solves word problems, products to 81.
LEVEL XIV	11-43-03-04	Timed test 36 products to 81 in one minute.
LEVEL XV	11-43-04-03 11-43-04-04	Finds products: 1 digit number, x any whole number.
	11-45-01-01	Selects missing digit; multiply 1 digit x any whole number.

CONTENT AREA: ALGORITHMS (Division)

Level	Number	Objective
LEVEL I	11-51-01-01	Partitions a set when given specified number in set.
	11-51-01-02	Partitions set when given specified number of subsets.
LEVEL VI	11-41-01-01	Identifies N sets with M objects, (total ≤ 25).
	11-51-01-01 11-51-01-03	Uses pictures to interpret division (total ≤ 25).
	11-51-01-04	Partitions set into N subsets of N with M elements (total ≤ 25).
	11-51-01-05	Partitions sets with number line drawings (total ≤ 25).
LEVEL X	11-51-01-01/02/03	Given equivalent subsets, reports ___ subsets of ___ with ___ remaining.
	11-51-01-04	Forms equivalent subsets; reports ___ subsets of ___ with ___ remaining.
	11-51-01-05	Uses number line to report: N = ____ sets of ____.
	11-51-01-06	Matches repeated subtraction to division products ≤ 25.
	11-51-02-01a 11-51-02-02a 11-53-02-01	Writes missing factor; form a ÷ b. Products ≤ 25.
	11-53-02-02	Writes missing _____ factor; form b⟌a. Products ≤ 25.
	11-53-02-03	Timed test; missing factor; a ÷ b or b⟌a. Products ≤ 25.
	11-52-01-01	Division properties of 0 and 1.
	11-53-02-04	Writes missing terms; form a ÷ b. Products ≤ 25.
	11-53-02-05	Writes missing terms; form b⟌a. Products ≤ 25.
	11-53-02-06	Timed test; missing terms; a ÷ b or b⟌a. Products ≤ 25.
	11-54-01-01	Writes equations and solutions; division word problems. Products ≤25.
	11-62-02-01 11-62-02-02	Matches division equations to multiplication equations. Products ≤ 25.

CONTENT AREA: **ALGORITHMS (Division)**

Level	Number	Objective
LEVEL XII	11-51-01-01/02/03	Express partitioning of sets (26-45) as: N = ____ subsets of ____ with ____ remaining:
	11-51-01-04	Partition sets (26-45) as: N = ____ subsets of ____ with ____ remaining.
	11-51-01-05	Partitioning line segments: (26-45) parts; complete statements.
	11-51-01-06	Matches repeated subtraction problems to division statements (26-45).
	11-53-02-01 11-53-02-02	Writes missing factor for division. Products (26-45).
	11-53-02-04 11-53-02-05	Writes missing numeral, division: (26-45).
	11-54-01-02	Writes equation and solves division word problems (26-45).
	11-62-02-04 11-62-02-05	Matches multiplication equations to related division equations and vice versa.
	31-53-04-01a	Divides money by whole number; cent notation, products ≤ 45¢.
	31-53-04-01b	Divides money by whole number; dollar notation, products ≤ $.45.
LEVEL XIII	11-53-02-03 11-53-02-06	Missing terms; division products 26-45, 1 minuted timed.
	11-51-01-01/02/03	Writes ___ subsets ___ with (remaining) = N. from picture. (products 45-81).
	11-51-01-04	Partition sets and writes ___ subsets of ___ with ___ (remaining) = N. (products 45-81).
	11-51-01-05	Writes ___ jumps of ___ with ___ remaining = N. from number line (46-81).
	11-51-01-06	Matches repeated subtraction to division; products. (46 - 81).
	11-51-02-01 11-51-02-02	Writes forms for division $\sqrt{}$, ÷, etc.
	11-53-02-01/02/03/04/05	Writes missing term, division, products. (46 - 81).

CONTENT AREA: **ALGORITHMS (Division)**

Level	Number	Objective
LEVEL XIV	11-53-02-03	Timed test. Supplies missing factors for division (46 - 81) one minute.
	11-53-02-06	Timed test. Supplies missing terms, 15 division exercises (46 - 81) one minute.
LEVEL XV	11-53-04-01	Divides: 3-digit dividend, 1 digit divisor, zero remainder.
	11-53-04-02	Divides: 3-digit dividend, 1 digit divisor, one rename, zero remainder.
	11-53-04-03	Divides any number by 1 digit divisor; zero remainder, zero(s) in dividend.
	11-53-04-04 11-53-04-05	Divides any number by 1 digit divisor; non-zero remainder.
	11-54-01-03	Solves division word problems products ≤ 81.

APPENDIX C

Contents: Survey form for teachers' perceptions of the difficulty of systems objectives dealing with number, numeration and basic operations.

Teacher Grade	Levels Responded To:
Kindergarten	1 and 2
First	3, 4 and 5
Second	6, 7 and 8
Third	9, 10 and 11
Fourth	12, 13 and 14

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL I
1	2	3	4	5	1. Identifies sets, equal or unequal in number.
1	2	3	4	5	2. Identifies sets: more, fewer, same number of elements.
1	2	3	4	5	3. Identifies sets, equal, unequal in number.
1	2	3	4	5	4. Makes second set equal in number to first set.
1	2	3	4	5	5. Makes non equivalent sets equivalent.
1	2	3	4	5	6. Learner selects counters to show number of things to 10.
1	2	3	4	5	7. Learner selects numeral cards to show number of things to 10.
1	2	3	4	5	8. Places pictures in specific ordinal positions.
1	2	3	4	5	9. Orders sets, smallest number to largest number.
1	2	3	4	5	10. Identifies cardinal number of 2 sets and cardinal number of union of 2 sets.
1	2	3	4	5	11. Separates sets, identifies cardinal number of subsets.
1	2	3	4	5	12. Identifies cardinal number of 2 sets, comparison.
1	2	3	4	5	13. Removes sets of specified cardinal number.
1	2	3	4	5	14. Partitions a set when given specified number in set.
1	2	3	4	5	15. Partitions a set when given specified number of subsets.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL II
1	2	3	4	5	1. Identifies set having one more or one less, 0 to 5.
1	2	3	4	5	2. Identifies set having one more or one less, 6 to 10.
1	2	3	4	5	3. Establishes set equivalence.
1	2	3	4	5	4. Matches set with cardinal number 0 to 5.
1	2	3	4	5	5. Matches set with cardinal number 6 to 10.
1	2	3	4	5	8. Identifies numeral that names greater number (10 or less).
1	2	3	4	5	9. Identifies numeral that names smaller number.
1	2	3	4	5	10. Identifies numeral that names the largest (greatest) number.
1	2	3	4	5	20. Identifies, penny, nickel, dime.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL III
1	2	3	4	5	1. Writes standard numeral (0-5).
1	2	3	4	5	2. Writes standard numeral (6-10).
1	2	3	4	5	3. Matches numeral to collection (11-20).
1	2	3	4	5	4. Writes standard numeral (11-20).
1	2	3	4	5	5. Writes standard numeral for written word names zero to ten.
1	2	3	4	5	6. Marks object in ordinal position, (first to fifth).
1	2	3	4	5	7. Matches addition equations to pictured union, sums to 10.
1	2	3	4	5	8. Writes addition equation from number line, sums to 10.
1	2	3	4	5	9. Writes equations to addition situations, sums to 10.
1	2	3	4	5	10. Writes the sum of 2 addends, horizontal and vertical, sums to 10.
1	2	3	4	5	11. Writes missing term (sum or addend), sums to 10.
1	2	3	4	5	12. Matches number line to subtraction sentence (sums to 10).
1	2	3	4	5	13. Matches picture to related subtraction sentence. (take away)
1	2	3	4	5	14. Matches picture to related subtraction sentence. (comparative)
1	2	3	4	5	15. Translates number line representation into equations, sums to 10.
1	2	3	4	5	16. Writes differences, vertical and horizontal form, sums to 10.
1	2	3	4	5	17. Writes missing numerals (sum or addend) subtraction, sums to 10.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Difficult	
					LEVEL IV
1	2	3	4	5	1. Horizontal form, sums to 10, one-minute timed test. (20 problems)
1	2	3	4	5	2. Vertical form, sums to 10, one-minute timed test. (20 problems)
1	2	3	4	5	3. Horizontal forms, differences to 10, one-minute timed test. (20 problems)
1	2	3	4	5	4. Vertical forms, differences to 10, one-minute timed test. (20 problems)
1	2	3	4	5	5. Identifies N sets of M number of objects; N ≤ 12.
1	2	3	4	5	6. Partitions objects into equivalent subsets of a given cardinal number. (number ≤ 12)
1	2	3	4	5	7. Partitions objects into a specified number of equivalent subsets. (number ≤ 12)
1	2	3	4	5	14. Draws hour on clock.
1	2	3	4	5	15. Draws hour hand on clock for "half-past" _____.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL V
1	2	3	4	5	1. Write standard numeral from pictured collection (to 50).
1	2	3	4	5	2. Matches ordinals to object in positional order (first through fifth).
1	2	3	4	5	3. Labels ordinal position of pictured events.
1	2	3	4	5	4. Counts by two's (to 18), ten's (to 50).
1	2	3	4	5	5. Writes expanded numeral from pictures, tens and ones (to 99).
1	2	3	4	5	6. Selects standard numeral from picture (to 99).
1	2	3	4	5	7. Writes standard numeral for standard expanded form.
1	2	3	4	5	8. Writes expanded form for standard form.
1	2	3	4	5	9. Matches addition number sentences; pictorial representations used, sums 11 to 18.
1	2	3	4	5	10. Writes number sentences, pictorial representations used, sums to 10.
1	2	3	4	5	11. Writes number sentences, pictorial representations used, sums 11 to 18.
1	2	3	4	5	12. Writes number sentences from number line (sums 11 to 18).
1	2	3	4	5	13. Translates oral statement into an equation; sums to 18.
1	2	3	4	5	14. Identifies "plus", "addend", "sum", "equals", or "add".
1	2	3	4	5	15. Groups addends by two's to find sums to 10.
1	2	3	4	5	16. Groups addends by two's to find sums to 18.
1	2	3	4	5	17. Finds sums (up to 4 addends) 1 digit addends, (sums to 10).

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL V (Continued)
1	2	3	4	5	18. Completes addition matrix, sums to 10.
1	2	3	4	5	19. Finds missing terms, (vertical form) (up to 4 addends) (sums to 10).
1	2	3	4	5	20. Sums to 18; 2 addends, horizontal form.
1	2	3	4	5	21. Sums to 18; 2 addends, vertical form.
1	2	3	4	5	22. Identifies "minus", "equal", "subtract", "minuend", "subtrahend", "difference", "sum", or "addend".
1	2	3	4	5	23. Horizontal form; sums to 18, supplies differences.
1	2	3	4	5	24. Vertical form, sums to 18, supplies differences.
1	2	3	4	5	25. Horizontal and vertical forms sums to 18, supplies differences.
1	2	3	4	5	26. Writes missing terms, horizontal and vertical forms--sums 11 to 18.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL VI
1	2	3	4	5	1. Matches words to specific parts of addition equation (add, plus, equals, sums, addend).
1	2	3	4	5	2. Matches addition word problem to equation, (sums to 10).
1	2	3	4	5	3. Write addition equation for word problem to equation, sums to 10.
1	2	3	4	5	4. Write equation and solution for word problems, sums thru 18.
1	2	3	4	5	5. Read word problem and write solution (sums to 10).
1	2	3	4	5	6. Read and write solution sums to 18.
1	2	3	4	5	7. Translates problems into subtraction story problems, (sums thru 10).
1	2	3	4	5	8. Match story problem to corresponding equations, (sums thru 18).
1	2	3	4	5	9. Matches subtraction word problem to equation (sums 11 to 18).
1	2	3	4	5	10. Identifies $\underline{N}$ sets with $\underline{M}$ objects, (total ≤ 25).
1	2	3	4	5	11. Uses pictures to interpret division, (total ≤ 25).
1	2	3	4	5	12. Partitions set into $\underline{N}$ subsets of $\underline{N}$ with $\underline{M}$ elements (total ≤ 25).
1	2	3	4	5	13. Partitions sets with number line drawings (total ≤ 25).
1	2	3	4	5	14. Uses $=$ or $\neq$ in number sentences.
1	2	3	4	5	15. Uses $<$ or $>$ to describe order relation between pairs.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	
					LEVEL VI (Continued)
1	2	3	4	5	18. Identifies quarters and half-dollars.
1	2	3	4	5	19. Learner marks coins given in cents.
1	2	3	4	5	20. Finds different ways to make nickel; dime.
1	2	3	4	5	21. Find different ways to make quarter.
1	2	3	4	5	22. Make clock show __:00 o'clock.
1	2	3	4	5	23. Makes clock show __:30 o'clock.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL VII
1	2	3	4	5	1. Matches ordinal words to given positions; 6th through 10th).
1	2	3	4	5	2. Counts by 3's to 18, counts by 5's to 20.
1	2	3	4	5	3. Giving cardinal number of sets grouped by 50's, 100's (200-1000).
1	2	3	4	5	4. Supplies expanded numeral from pictures: (100-999).
1	2	3	4	5	5. Interprets positional value of specified digits, 3-digit numerals.
1	2	3	4	5	6. Selects or writes standard form or other expanded form, for 3-digit numerals.
1	2	3	4	5	7. Matches examples of commutative property to number lines.
1	2	3	4	5	8. Gives cardinal number of the union of sets as 10 plus ___, (sums 11-18).
1	2	3	4	5	9. Writes equations: form a + b = b + a, using number lines.
1	2	3	4	5	10. Completes addition equations, associative property, using number lines.
1	2	3	4	5	11. Applies associative property (sums to 18).
1	2	3	4	5	12. Completes addition matrices; sums 11-18.
1	2	3	4	5	13. Addition equations; 2, 3, 4 addends; missing terms; sums 11-18; horizontal.
1	2	3	4	5	14. Addition equations; 2, 3, 4 addends; missing terms; sums 11-18, vertical.
1	2	3	4	5	15. One minute timed test; horizontal, sums 11-18.
1	2	3	4	5	16. One minute timed test; vertical, sums 11-18.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	
					LEVEL VII (Continued)
1	2	3	4	5	17. Vertical form; 3-4-5 digit addends--sums to 18.
1	2	3	4	5	18. Sums, 1 and 2-digit addends; horizontal and vertical; no regrouping.
1	2	3	4	5	19. Sums, 1 and 2-digit addends; horizontal and vertical; regrouping.
1	2	3	4	5	20. Sums, 2 or 3-digit addends, vertical; no regrouping.
1	2	3	4	5	21. Subtraction as noncommutative operation; number lines.
1	2	3	4	5	22. One minute timed test; subtraction; horizontal, sums 11-18.
1	2	3	4	5	23. One minute timed test, subtraction; vertical, sums 11-18.
1	2	3	4	5	24. Supplies differences 1 and 2 digit; vertical, no regrouping.
1	2	3	4	5	25. Supplies differences 2 and 3-digit numerals; no regrouping.
1	2	3	4	5	26. Supplies missing terms, subtraction; 1 and 2-digit; no regrouping.
1	2	3	4	5	27. Supplies missing terms, subtraction; 2 and 3-digit; horizontal, no regrouping.
1	2	3	4	5	28. Supplies missing terms, subtraction; up to 3 digits; horizontal and vertical; no regrouping.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL VIII
1	2	3	4	5	1. Writes equation for word problems, sums through 18.
1	2	3	4	5	2. Solves word problems; sums 11-18.
1	2	3	4	5	3. Interprets multiplication; identifies factor set, and completes equations. Products $\leq$ 25.
1	2	3	4	5	4. Translates word problems into equations and solves products $\leq$ 25.
1	2	3	4	5	5. Selects subtraction equations from addition equations. Sums $\leq$ 10.
1	2	3	4	5	6. Selects addition equations from subtraction equations. Sums $\leq$ 10.
1	2	3	4	5	7. Selects subtraction equations from addition equations. Sums 11-18.
1	2	3	4	5	8. Selects addition equations from subtraction equations; sums 11-18.
1	2	3	4	5	9. Identifies larger or smaller number: 11-100.
1	2	3	4	5	10. Identifies smallest or greatest number in a triplet of numbers: 11-100.
1	2	3	4	5	11. Uses = or $\neq$ to describe related numbers: 11-100.
1	2	3	4	5	12. Uses $>$ or $<$ to relate numbers: 11-100.
1	2	3	4	5	13. Writes unit fractions from unit wholes partitioned into 2,3,4,5, or 6 congruent parts.
1	2	3	4	5	17. Matches pictures of coins to their values in cent notation: $\leq$ 25¢.
1	2	3	4	5	18. Matches pictures of coins to written words: values to 25¢.
1	2	3	4	5	19. Matches pictures of coins to value in dollar notation: $\leq$ 25¢.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	
					LEVEL VIII (Continued)
1	2	3	4	5	20. Writes values of pictured coins in both cent and dollar notation: ≤ 25¢.
1	2	3	4	5	21. Writes sums in cent notation: sums ≤ 10¢.
1	2	3	4	5	22. Writes sums in cent notation: sums ≤ 20¢.
1	2	3	4	5	23. Writes total value of pictured items labeled as to value; value ≤ 10¢.
1	2	3	4	5	24. Finds value of selected items and amount of change. Value ≤ 10¢.
1	2	3	4	5	25. Arranges written names of the month in order.
1	2	3	4	5	28. Writes differences in cent notation: sums ≤ 10¢.
1	2	3	4	5	29. Writes differences in cent notation: sums ≤ 20¢.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL IX
1	2	3	4	5	1. Writes standard numerals for written word names; 11-99.
1	2	3	4	5	2. Writes standard numerals for written word names, 100-1000.
1	2	3	4	5	3. Reads written ordinal words and matches to pictured ordered items in those positions, 11th to 20th.
1	2	3	4	5	4. Matches ordinal abbreviations to ordered pictured items, 1st to 19th.
1	2	3	4	5	5. Selects appropriate written ordinal word names for ordinal numbers, 25 to 100.
1	2	3	4	5	7. Writes addition equations from verbal descriptions of additive situations.
1	2	3	4	5	8. Determines number by which one set has more or fewer items and selects appropriate number sentences.
1	2	3	4	5	9. Supplies or selects number sentences for pictured set subtraction scenes.
1	2	3	4	5	10. Writes word problems for pictured subtraction situations.
1	2	3	4	5	11. Selects subtraction story problems for a-b=c, $a \leq 18$, with a, b, or c as a place holder.
1	2	3	4	5	12. Reads written subtraction word problems with examiner and solves; sums to 10.
1	2	3	4	5	13. Reads written subtraction word problems with examiner and solves; sums 11-18.
1	2	3	4	5	14. Supplies expressions from pictured set situations; a x b, products ≤ 25.
1	2	3	4	5	15. Interprets expressions, horizontal form, $a_1 + a_3 + \ldots + a_n$ = n sets of a or n • a, product ≤ 25.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL IX (Continued)
1	2	3	4	5	16. Interprets expressions, vertical form, $a_1 + a_2 + a_3 + \ldots + a_n$ = n sets of a or n · a, products ≤ 25.
1	2	3	4	5	17. Writes multiplication and addition number sentences; products and sums ≤ 25.
1	2	3	4	5	18. Supplies number names of equal addends and related factors from number lines showing repeated addition.
1	2	3	4	5	19. Supplies multiplication sentences from arrays.
1	2	3	4	5	20. Uses multiplication to name the number of possible pairings between elements of two sets, # of pairings ≤ 25.
1	2	3	4	5	21. Organizes pairings of 2 sets to show multiplicative situations ≤ 25.
1	2	3	4	5	22. Identifies the multiplication properties of 1 and 0 by writing products or completing generalizations concerning the effect 1 and 0 have upon products.
1	2	3	4	5	23. Uses commutative and associative properties to complete multiplication number sentence and matrix.
1	2	3	4	5	24. Writes products in horizontal and vertical form, products to 25.
1	2	3	4	5	25. Completes multiplication sentences and matrix, products to 25.
1	2	3	4	5	26. Two-minute timed test, products to 25, vertical form.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL X
1	2	3	4	5	1. Multiplication word problems, equations and solutions: Products ≤ 25.
1	2	3	4	5	2. Given equivalent subsets, reports __ subsets of __ with __ remaining.
1	2	3	4	5	3. Forms equivalent subsets. Reports __ subsets of __ with __ remaining.
1	2	3	4	5	4. Uses number line to report: N = __ sets of __.
1	2	3	4	5	5. Matches repeated subtraction to division, products ≤ 25.
1	2	3	4	5	6. Writes missing factor; form $a \div b$. Products ≤ 25.
1	2	3	4	5	7. Writes missing factor; form $b\overline{)a}$. Products ≤ 25.
1	2	3	4	5	8. Timed test; missing factor; $a \div b$ or $b\overline{)a}$. Products ≤ 25.
1	2	3	4	5	9. Division properties of 0 and 1.
1	2	3	4	5	10. Writes missing terms; form $a \div b$. Products ≤ 25.
1	2	3	4	5	11. Writes missing terms; form $b\overline{)a}$. Products ≤ 25.
1	2	3	4	5	12. Timed test; missing terms; $a \div b$ or $b\overline{)a}$. Products ≤ 25.
1	2	3	4	5	13. Writes equations and solutions; division word problems. Products ≤ 25.
1	2	3	4	5	14. Matches subtraction equations to addition equations. Sums ≤ 100.
1	2	3	4	5	15. Matches addition equations to subtraction equations. Sums ≤ 100.
1	2	3	4	5	16. Matches division equations to multiplication equations. Products ≤ 25.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	
					LEVEL X (Continued)
1	2	3	4	5	17. Matches multiplication equations to division equations. Products $\leq$ 25.
1	2	3	4	5	18. Writes unit fractions; using regions 1/7-1/10.
1	2	3	4	5	19. Writes unit fractions; using number line 1/2 - 1/5.
1	2	3	4	5	21. Identifies coin combinations equivalent to quarter.
1	2	3	4	5	22. Writes total value of labeled items. Value $\leq$ 25¢.
1	2	3	4	5	23. Selects items; finds total value. Writes change. Values $\leq$ 25¢.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL XI
1	2	3	4	5	1. Matches numeral to written word name.
1	2	3	4	5	2. Writes standard numeral for picture (1000-3000).
1	2	3	4	5	3. Writes expanded form, (100-999).
1	2	3	4	5	5. Sums 2 or 3 digit addends, vertical; regrouping.
1	2	3	4	5	6. Matches word problem to addition equation, sums to 100.
1	2	3	4	5	7. Writes equation for addition word problems; sums to 100.
1	2	3	4	5	8. Writes solutions for addition word problems sums (100-1000).
1	2	3	4	5	9. Reads and solves addition word problems, sums $\leq$1000.
1	2	3	4	5	10. Selects best minuend, regrouping tens to ones; hundreds to tens; 2 and 3 digits.
1	2	3	4	5	11. States whether regrouping is needed: 2 and 3 digit subtraction problems.
1	2	3	4	5	12. Relates zero concepts to subtraction problems (T. or F.).
1	2	3	4	5	13. Selects equations for subtraction word problem; 2 and 3 digits; no regrouping.
1	2	3	4	5	14. Equations which are not translations of subtraction word problem; 2 and 3 digits, no regrouping.
1	2	3	4	5	15. Solves subtraction word problems, sums (18-99); no regrouping.
1	2	3	4	5	16. Multiplication facts from pictures; products 26 - 45.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	
					LEVEL XI (Continued)
1	2	3	4	5	17. Product expressions from pictures 26 - 45.
1	2	3	4	5	18. Addition and multiplication equations from pictures (26-45)
1	2	3	4	5	19. Addition and multiplication equations from number lines (26 - 45).
1	2	3	4	5	20. Multiplication equation from rectangular arrays (26 - 45).
1	2	3	4	5	21. Matches words to symbols in addition and multiplication equations. (26 - 45)
1	2	3	4	5	22. Multiplication properties of 1 and 0 (products 26 - 45).
1	2	3	4	5	23. Complete multiplication matrix, products to 45.
1	2	3	4	5	24. Products through 45, horizontal and vertical.
1	2	3	4	5	25. Matches number to its multiples to 45.
1	2	3	4	5	26. Writes equation and solves multiplication word problems; products to 45.
1	2	3	4	5	27. Matches coins to values in cents ($\leq$ 50¢).
1	2	3	4	5	28. Matches coins to written word value, half-dollar.
1	2	3	4	5	29. Matches coins to values; dollar notation ($\leq$ $.50).
1	2	3	4	5	30. Writes value of coins to 50 cents; <u>cent</u> and <u>dollar</u> notation.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL XII
1	2	3	4	5	1. Products: 1 minute timed test; products $\leq$ 45.
1	2	3	4	5	2. Express partitioning of sets (26-45) as: N = __ subsets of __ with __ remaining.
1	2	3	4	5	3. Partitions sets (26-45) as: N = __ subsets of __ with __ remaining.
1	2	3	4	5	4. Partitioning line segments: (26-45) parts; complete statements.
1	2	3	4	5	5. Matches repeated subtraction problems to division statements. (26-45)
1	2	3	4	5	6. Writes missing factor for division: products (26-45).
1	2	3	4	5	7. Writes missing numeral, division: (26-45).
1	2	3	4	5	8. Writes equation and solves division word problems (26-45).
1	2	3	4	5	9. Matches multiplication equations to related division equations and vice versa.
1	2	3	4	5	10. Shades $\frac{2}{3}$, $\frac{2}{4}$, $\frac{3}{4}$ of unit whole.
1	2	3	4	5	11. Circles $\frac{2}{4}$, $\frac{3}{4}$, $\frac{2}{3}$ of sets 4, 6, 8, 9, 12 or 16 items.
1	2	3	4	5	12. States numbers of parts needed to make one whole.
1	2	3	4	5	13. Shades fractional parts of regions: 5 to 10 congruent parts.
1	2	3	4	5	14. Identifies <u>numerator</u> and <u>denominator</u>, matches definitions to corresponding parts.
1	2	3	4	5	20. Combines coins to produce 50¢.
1	2	3	4	5	21. Purchases; fewest coins. Total $\leq$ $1.00.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	
					LEVEL XII (Continued)
1	2	3	4	5	22. Adds money, dollars and cents, horizontal and vertical.
1	2	3	4	5	23. Subtracts money, cent notation, no regrouping, sums ≤ $1.00.
1	2	3	4	5	24. Subtracts money, dollar and cents, no regrouping, sums ≤ $10.
1	2	3	4	5	25. Multiplies money by whole number; dollar notation, product ≤ 45¢.
1	2	3	4	5	26. Multiplies money by whole number; dollar notation, product ≤ $.45.
1	2	3	4	5	27. Divides money by whole number; cent notation, product ≤ 45¢.
1	2	3	4	5	28. Divides money by whole number; dollar notation, product ≤ $.45.
1	2	3	4	5	29. Identifies odd and even numbers; ≤ 100.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL XIII
1	2	3	4	5	1. Supplies numeral for written number. ≤ 1000.
1	2	3	4	5	2. Writes standard numeral or expanded form for written form.
1	2	3	4	5	3. Writes standard numeral or expanded form for $(a \times 10) + b$ or $(a \times 0) + b \cdot a \leq 9$, $b \leq 9$.
1	2	3	4	5	4. Rounds 2-digit numerals to nearer ten.
1	2	3	4	5	5. Missing terms, 2-3 digit addition, regrouping.
1	2	3	4	5	6. Mis ing terms, 2-3 digit subtraction, no regrouping.
1	2	3	4	5	7. Finds differences, 2-3 digit subtraction, no regrouping.
1	2	3	4	5	8. Finds differences, 1,2,3 digit subtraction, regrouping.
1	2	3	4	5	9. Missing terms, 2-3 digit subtraction, regrouping.
1	2	3	4	5	10. Subtraction word problems, sums 18 to 99, regrouping.
1	2	3	4	5	11. Matches expressions to picture products, 46-81.
1	2	3	4	5	12. Matches products to repeated addition products, 46-81.
1	2	3	4	5	13. Writes addition and multiplication equations from pictures, products 46-81.
1	2	3	4	5	14. Number lines, repeated addition and multiplication. Products 46-81.
1	2	3	4	5	15. Writes multiplication equation from arrays. Products 46-81.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	
					LEVEL XIII (Continued)
1	2	3	4	5	16. Missing terms from multiplication facts to 81.
1	2	3	4	5	17. Writes products to 81.
1	2	3	4	5	18. Writes multiplication expression expressions for multiples of N. (46-81)
1	2	3	4	5	19. Names multiples of N; Identifies N from a set of multiples of N.
1	2	3	4	5	20. Writes equation and solves word problems, products to 81.
1	2	3	4	5	21. Missing terms; division products 26-45, 1 minute timed.
1	2	3	4	5	22. Writes __ subsets of __ with __ (remaining) = N. from picture. (products 45-81).
1	2	3	4	5	23. Partitions sets and writes __ subsets of __ with __ (remaining) + N. (products 45-81).
1	2	3	4	5	24. Writes __ jumps of __ with __ remaining = N. from number line (46-81).
1	2	3	4	5	25. Matches repeated subtraction to division; products (46-81).
1	2	3	4	5	26. Writes forms for division. $\overline{)}$, $\div$, etc.
1	2	3	4	5	27. Writes missing term, division, products. (46-81)
1	2	3	4	5	28. Subtracts money, cent notation, to 99¢ with regrouping.
1	2	3	4	5	29. Classifies numbers to 100 as prime, composites, or neither.
1	2	3	4	5	30. Labels statements about primes and composites as T. or F.
1	2	3	4	5	31. Classifies sums as odd or even.
1	2	3	4	5	32. Classifies products as odd or even.

Directions: Mark out the number which best indicates your feeling of the difficulty level for each objective.

Extremely Difficult	Mildly Difficult	Average	Relatively Easy	Extremely Easy	LEVEL XIV
1	2	3	4	5	1. Timed Test: 36 products to 81 in one minute.
1	2	3	4	5	2. Timed Test: Supplies missing factors for division (46-81) one minute.
1	2	3	4	5	3. Timed Test: Supplies missing terms, 15 division exercises; (46-81), one minute.
1	2	3	4	5	6. Shades parts from fraction word names.
1	2	3	4	5	7. Shades parts from fractional numerals.
1	2	3	4	5	8. Writes fractions to name shaded parts.
1	2	3	4	5	9. Writes fractions from number line.
1	2	3	4	5	28. Identifies fraction names for one; states multiplication property of 1.

APPENDIX D

Contents: Copy of article sent to Instructor magazine.

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TEACHING NUMERATION: PIECING THE PUZZLE TOGETHER

by

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and
Dr. Gilbert J. Cuevas

The teaching of our numeration system is considered an integral part of the primary mathematics curriculum. Numeration is developed along with the concept of number and the operations of addition, subtraction and multiplication in the first three grades of school. It is expanded as the child moves through the grades, until the repeating pattern for reading large numerals has been mastered. The system is regarded as a simple, precise way of expressing numbers; it uses only ten symbols, each having a precise value and a place value.

The problem for young children is that while our numeration system is simple and precise, it is also extremely abstract. This abstract quality presents problems to the young learner attempting to master the concepts involved in numeration.

Because of the difficulties children have in mastering numeration objectives, the authors felt a content analysis of existing math programs should be made to determine specifically what is being taught about numeration. The content analysis centered on two areas: First, the existing literature on the teaching of mathematics was surveyed to ascertain what areas should be taught in numeration; second, objectives from various systems management programs for mathematics were examined to determine what objectives are taught under the heading of numeration. The two findings were then compared.

Review of Existing Literature on Teaching Mathematics

Literature on the teaching of mathematics deals with the basic theme of our numeration system, its additive nature. A number, for example, 524, can be expressed as $500 + 20 + 4$, or as $(5 \times 10^2) + (2 \times 10^1) + (4 \times 10^0)$, or in various other combinations. But regardless of the nature of the expanded notation, the value of the numeral is obtained by adding the values for each position together.

The other major idea taken from mathematics literature was that there are five basic components necessary for the development of our numeration system. They are:

1. The ten one-digit numerals, 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These are the symbols of the system, the basic building blocks used. Each symbol may imply either an absolute value or a value coupled with the place value of the position in which the numeral occurs.

2. The concept of zero: In the historical development of our numeration system zero was the final concept developed. The idea that you need a symbol to represent nothing is fairly advanced; it needs special development with young children.

3. A base: The base used for a numeration system determines the size of the sets used for grouping within the system. Our decimal base system indicates that we will use groups of ten or powers of ten for our grouping procedure.

4. Place value: The central concept of the numeration system is that numerals occupying different positions in the system have different values. It is the teaching of the place value of each position in a numeral which presently occupies most of our efforts in teaching numeration.

5. The decimal point: This is used to extend the place value system into fractional numerals. The teaching of this concept is usually left to the intermediate grades; it will not be dealt with further in this article.

Review of Systems Management Programs for Mathematics

The authors also examined objectives from various systems management programs to determine what is actually being taught under

the heading of numeration. Both commercially developed management systems and those developed by local school districts were examined. Since both program types contained similar objectives, no further differentiation will be made between them. A sampling of the objectives found on numeration is given in Table One. The table, while omitting repetitious objectives, combining others and covering only two-digit numerals illustrates the range of numeration objectives identified.

TABLE ONE

A Sampling of Numeration Objectives
From Systems Management Programs in Mathematics

Objective	Concept Being Taught
The child will be able to:	
1. identify sets equal, unequal in number.	Number Equivalence
2. select counters to show number of objects to 10.	Number
3. write standard numerals, 1 - 9, 10 - 99.	Numerals
4. identify numerals that name smaller/larger number.	Number/numeral
5. match numeral to a picture collection.	Number/numeral
6. identify N sets of M number of objects.	Base
7. count by tens to 100.	Base
8. write expanded numerals from pictures, 10's and 1's.	Base
9. identify numerals in tens and ones place.	Place Value
10. write expanded numerals from pictures, to 99.	Place Value
11. write the standard numeral from expanded forms.	Place Value

As we can see from the objectives presented, a certain ordered sequence of objectives is apparent. Instruction begins with the development of the concept of number, states the relationship between number and numeral, moves on to the writing of numerals, then develops the concept of base, and finally deals with place value. Notice that objectives dealing with zero are not apparent; they did not appear specifically in the systems management programs. Zero was treated as any other numeral. The authors, however, feel that zero should be given special emphasis because of its important role as a place holder in our numeration system.

The primary purpose for examining the content suggestions and requirements for numeration was to obtain suggestions as to how the concept of numeration is currently being presented in the primary grades. The authors feel that numeration should be approached in the manner in which a child would solve a puzzle. The parts are first handled and understood (numerals, zero, base), operations or rules for placing the parts together are practiced (place value, addition, multiplication) and at the end the student has the satisfaction of being able to construct the complete picture (the numeration system).

Let us turn, then, to the teaching of numeration. What suggestions would we make to the teacher who is attempting to present a comprehensive program to students in order to teach numeration? An obvious suggestion is that all the components of our numeration system should be taught. So let us group the suggestions under the headings of the four component areas important to learning numeration in the primary grades.

Objectives Dealing With Learning of Numerals

1. The child will memorize the order of the counting numerals. This implies that exercises in rote counting will be given to the child; the following order is suggested for such exercises:

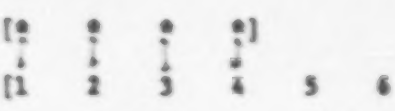
a. First, from one to ten, then eleven to twenty. This activity teaches the child the numeral names. The first ten digits will be used over and over; the second ten are slightly different and must be memorized separately.

b. The child will count by tens from ten to one hundred. Children often make the mistake in counting of going "... twenty-eight, twenty-nine, twenty-ten, twenty-eleven, etc." Or they make a corollary mistake, moving from "... twenty-eight, twenty-nine, (pause)..." In both cases the child is having a similar problem: he does not know the name for or the beginning point for each new group of ten. The process of counting by tens teaches the child the numeral names for each new grouping. Then when the child learns the repeating pattern of counting from twenty to twenty-nine, thirty to thirty-nine, etc., he will have the name for the next group of ten in his counting sequence.

c. The child will count from one to one hundred in order. The final counting objective is for the child to demonstrate his mastery of the entire pattern of numerals, 0 - 100. With this information he will be able to work all place value exercises of two digits.

2. The child will write the basic numerals, 0 - 9. As the child learned the names for numerals, so he must also learn the symbols for the numerals. This is one of the few psycho-motor domain objectives for primary mathematics education and should be taught as such. The child needs physical practice in forming the numerals, ranging from practice writing large numerals in the air or tracing sandpaper numerals down to the actual process of writing each numeral correctly on paper.

3. The child will match the numeral to a set containing the proper amount of elements. This objective connects number with numeral, symbol with amount. It emphasizes the child's ability to "count" sets. The child can either count the elements of a set or he can pair the elements with the set of counting numerals, as in the following example:



The last element paired, then, counts the set. Regardless of whether the child is able to count the set or if he must pair the elements of the set to the counting numerals, the final goal is the same. He matches number with numeral.

Objectives Dealing With the Concept of Zero

4. The child will match the numeral zero to an empty set. We have stated earlier that the numeral zero needs to have special attention paid to it. The child should be asked to construct a set with no elements, pair that set with the numeral representing that set, and

finally, to discuss just what the concept of zero means. In this manner the teacher can ascertain if the child is developing a proper concept of zero.

Objectives Dealing With the Concept of Base

5. The child will construct N sets of N elements. The first objective dealing with the concept of "base" should deal simply with the idea of grouping. Can the child make a group of three or four? Can he understand the process of constructing several groups of the same size? Since Piaget states that a child can understand smaller numerals at a younger age, the child's first grouping exercises should focus on making small groups, generally less than five. The child moves on to larger numerals as his reasoning capabilities expand. The teacher's final goal, of course, is to move to groups of ten.

Two observations should be made about such grouping exercises. First, the beginning exercises should always come out even. If the child is asked to make two groups of three elements, he should be given six objects to work with. The other possible alternative would be to give the child a large set and to ask him only to remove what items he needed. The second point is that remainders should be slowly added to the grouping exercises. Given seven objects, then, the child should be asked to make two sets of three objects and to explain what happened differently in this exercise. In this way the child is introduced to the fact that not all groupings work out evenly; he will sometimes have remainders. When the child finally works with groups of ten in our

place value system, he will then be used to the fact that there are usually objects left over when he makes groups of ten.

6. The child will construct groups of ten, with remainders. The skill of reading and writing two-digit numerals depends upon the child's ability to recognize that the numeral in the second position stands for groups of ten. Introductory objectives, then, should deal with the child and his ability to make groups of ten out of given sets. If the child is given a set of fourteen objects, for example, he should be able to show that he can construct one group of ten objects and has four left over.

Objectives Dealing With Place Value

7. The child will be able to write the numeral representing groups of ten with n remainders, n less than or equal to nine. Many math objectives have corollaries; objective #6 deals with making sets; objective #7 deals with writing the numeral representing each set. This enables the child to understand that whatever he can do with things, he can represent with numerals. He begins to pair concrete work with objects with abstract work with the symbols which represent those objects.

8. The child will build groups of ten with no remainders, in order to use a zero as a place holder. Again, we have the special emphasis upon zero. The child needs to analyse numbers such as twenty or thirty and indicate how such numerals can be written. He can see that two and twenty are different amounts; therefore, the symbols for two and twenty must be different.

9. The child will build groups of 100's, 10's, and 1's and write the symbols to represent those numerals. With this final objective, we indicate that the child can then move on to three, four, or greater number of digits by basically using the same objectives as indicated earlier. Such objectives simply extend the child's knowledge; they still use the same basic idea of pairing the writing of the numeral with the construction of the groups. In this way we continue to pair the concrete with the abstract for the child to see the connection between the two.

Conclusion

A step-by-step procedure which contains all the components of our number system is necessary in order for the child to obtain a thorough knowledge of our numeration system. Children who have developed such knowledge should be able to use and understand later concepts involving place value and numeration, such as expanded notation, regrouping in both addition and subtraction, or of placing the partial products in a two-digit multiplication problem. It is hoped that by taking a sequential components approach to the teaching of numeration, the student will reach a point where an "Aha!" response will signify his having a clear and correct idea regarding numeration concepts.